# Bayesian data analysis using JASP Dani Navarro



compcogscisydney.com/jasp-tute.html

## Part I:Theory

- Philosophy of probability
- Introducing Bayes rule
- Bayesian reasoning
- A simple example
- Bayesian hypothesis testing

### Part 2: Practice

- Introducing JASP
- Bayesian ANOVA
- Bayesian t-test
- Bayesian regression
- Bayesian contingency tables
- Bayesian binomial test

# I.I Philosophy of probability

## Idea #I: "Aleatory" processes



Probability is an objective characteristic associated with physical processes, defined by counting the relative frequencies of different kinds of events when that process is invoked

## "Aleatory" processes







## Frequentist statistics



Coin flipping is an aleatory process, and can be repeated as many times as you like



The probability of a head is defined as the long-run frequency

## Frequentist statistics



A particle physics experiment is a repeatable procedure, and thus a frequentist probability can be constructed to describe its outcomes



A scientific theory is <u>not</u> a repeatable procedure, and cannot be assigned a probability: there is no such thing as "the probability that my theory is true"

## Idea #2:"Epistemic" uncertainty

Probability is an subjective characteristic associated with rational agents, defined by assessing the strength of belief that the agent holds in different propositions





## "Bayesian" statistics



Probabilities can be attached to any proposition that an agent can believe A particle physics experiment generates observable events about which a rational agent might hold beliefs



A scientific theory contains a set of propositions about which a rational agent might hold beliefs

## 1.2 Introducing Bayes rule



Roll two dice...



#### Thirty six possible cases



#### Three cases where the dice add up to 4



#### All 36 cases organised by outcome







## A:"at least one die has a value of 2"



$$P(A) = \frac{11}{36} = .31$$

### B:"the total is at least six"



$$P(B) = \frac{26}{36} = .72$$

= 26/36

Probability that the total is at least 6

P(B)



 $(\mathbf{I})$ = 6/26= 26/36Probability that at least one die has a Probability that the total is at least 6 2 given that the total is at least 6 P(B) = P(A|B)P(A)Probability that at least one die has a 2 = 11/36

= 26/36Probability that the total is at least 6 P(A|B)P(B|A)P(A)Probability that the total is at least 6 given that at least one die has a 2 = 6/11  $(\bullet)$  $(\bullet)$ 

= 6/26

Probability that at least one die has a 2 given that the total is at least 6

Probability that at least one die has a 2





## Let's check that:



## Let's check that:



## **I.3** Bayesian reasoning

# **Bayes' rule** is a mathematical fact that probabilities must obey



**Bayesian reasoning** happens when we combine this mathematical rule with epistemic probability



## For example...



h = A hypothesis about the world

d = Some observable data



How strongly should I believe ... given that I have observed these data?



#### h|d



The **posterior probability** that my hypothesis is true given that I have observed these data...

$$P(h|d) = rac{P(d|h) imes P(h)}{P(d)}$$



# The **prior probability** that I assigned to this hypothesis before observing the data

$$P(h|d) = \frac{P(d|h) \times P(h)}{P(d)}$$





The <u>likelihood</u> that I would have observed these data if the hypothesis is true

$$P(h|d) = rac{P(d|h) imes P(h)}{P(d)}$$

 $P(h|d) = \frac{P(d|h) \times P(h)}{P(d)}$ 



The "marginal" probability of observing these particular data (more on this shortly)
## **Belief revision!**





P(d) : discussed later

## 1.4 Example of Bayesian reasoning



## Many possibilities



dropped a wine glass



broke a window



psychic explosion



earthquake



a wizard did it

etc...

## Let's compare two of them





I dropped a wine glass

Kids broke the window

### "Prior odds"

 $\frac{P(h_1)}{P(h_2)}$ 





### = 0.1

Before learning anything else I think "wine glass dropping" is 10 times more plausible than "broken window"

### Some data



## There is a cricket ball next to the broken glass

## Likelihood of the data

When I drop a wine glass...





... It's very unlikely that I just happen to do so right next to a cricket ball

P(d|h) = 0.001

## Likelihood of the data

When the kids break a window...





... It's not at all uncommon for a cricket ball to end up near the glass

P(d|h) = 0.15

## **Bayes** factor

(a.k.a. likelihood ratio)



I think it is 150 times more likely that I would find a cricket ball when a window breaks than when a wine glass is broken

## Posterior odds

 $\frac{P(h_1|d)}{P(h_2|d)} = \frac{P(d|h_1)}{P(d|h_2)} \times \frac{P(h_1)}{P(h_2)}$ 



Posterior odds = 15

Likelihood ratio = 150

Prior odds = .

In light of the evidence, I now think the windowbreaking hypothesis is 15 times more likely than the wine-glass hypothesis





## 1.5 Bayesian hypothesis testing





# Is this roulette wheel unbalanced?



We're ignoring the zero



<u>Null model,</u>

The roulette wheel has an equal probability of producing red and black

 $h_0$ 





<u>Null model,</u>

The roulette wheel has an equal probability of producing red and black

 $h_0$ 

<u>Alternative model</u>,  $h_1$ 

The roulette wheel has a bias, but we don't know what it is





Let's pretend that there's no such thing as "continuous numbers", and act as if the only possible values for P(red) are 0, 0.1, 0.2, ..., 1.0 ©





Likelihoods ... the probability of the data given every possible value of P(red)





The null hypothesis assigns prior probability 0 to the possibility that P(red) = 0.8 ... ... so even though it assigns highest likelihood to the observed data ....



The null hypothesis assigns prior probability I to the possibility that  $P(red) = 0.5 \dots$  ... so even though it assigns a pretty small likelihood to the observed data ....





# DataModels8 red $\swarrow$ Null model $h_0$ Bayes factorThe roulette wheel has<br/>an equal probability of<br/>producing red and black $P(d|h_0)$ ... evidence

<u>Alternative model</u>  $h_1$ 

2 black

The roulette wheel has a bias, but we don't know what it is ... evidence of about 2:1 in favour of the alternative

 $P(d|h_1)$ 

 $BF_{10} = \frac{P(d|h_1)}{P(d|h_0)} = \frac{\sum_{\theta} P(d|\theta) \times P(\theta|h_0)}{\sum_{\theta} P(d|\theta) \times P(\theta|h_1)} = 1.87$ 

### 2.1 Just another stats package https://jasp-stats.org



Home - JASP - Free Statistics: ×

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**JASP** DOWNLOAD WORKSHOPS | VIDEOS | TEACHING | BLOO HOME A Fresh Way to terms 1. 14. 100 近 % 4 **Do Statistics** framework (1) First and Pentalish characterization (sector) and in providing and 81.1238 -14 Francis Saluates, 6.4 - . Proje . -Download .... E-10.44 G-14-180 In section at a 8.41 a free takes Instantial Autority or Train sectors 10 the state of the s -0.04 Non-pile perginate p. 7 11.4 0.8.1.2 \*\* Bris print services 5 1 1000 **NEW RELEASE EPS** format -

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## Illustrating the JASP workflow

<u>What?</u>

open a CSV file

descriptive statisitics

run a frequentist ANOVA

save data and results to JASP file

Where?

File > Open

**Common > Descriptives** 

Common > ANOVA > ANOVA

File > Save As

### Here's a real data set with many variables!

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JASP isn't (currently?) good for computing new variables, so it's best to do that in Excel or whatever you prefer

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9	smallworld	property	3.17
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12	smallworld	category	5.50
13	smallworld	property	3.00
14	largeworld	property	4.83
16	smallworld	category	3.83
18	smallworld	category	4.50
19	smallworld	category	3.50
20	smallworld	property	5.50

For simplicity I'll use small CSV files with only the relevant variables

tutedata l.csv

### File > Open



### Common

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#### Common



#### File > Save As

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#### File > Export Results

## 2.2 Bayesian ANOVA



#### Common > ANOVA > Bayesian ANOVA



#### Common > ANOVA > Bayesian ANOVA

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4	4	largeworld	property	4.333	frequency		1.947	1	3,947	3.551	0.061		
6	5	smailworld	category	6	frequency + sampling	1	5.429	î	5.429	4.884	0.028		
	6	largeworld	property	5.1064	Residual	31	1.523 2	282	1.112	0.000			
7	7	smailworld	category	5.1061	Noor, Type III Sum of Squ	APAGE .							
		smallworld	property	6.033									
	0	amailworld	oroperty	3.1667	- Resident Allender								1
-	-	Internet	annually.	4	Bayesian ANOV	A							
-	~	angenore.	Property.		Model Comparison - mes	angen							
-		smanworld	category	3.000	Ma	dels.		(M)	PONTIdutal	10 <sup>1</sup> M	8F10	error N	
12	12	smallworld	category	9.9	Null model			0.200	1.195e -9	4.780e -9	1.000	-	
13	13	smallworld	property	3	frequency			0.200	3.823e-10	1.529e -9	0.320	1.317s -5 2.988s -14	
14	54	largeworld	property	4.833	frequency + sampling			0.200	0.255	1.367	2.132e+8	0.849	
15	56	smallworld	category	3.833	frequency + sampling	+ frequency	+ sampling	0.200	0.436	3.089	3.6476+8	1.130	
10	18	amailworld	category	4.5									
17	19	smallworld	category	3.5	Analysis of Effects - mea	ngen							
18	20	smailworld	property	5.5	Effects	Planci	Ponci data)	Pirela	sian				
19	21	smallworld	property	5.833	frequency sampling frequency + sampling	0.600 0.600 0.200	0.691 1.000 0.436	4.227e	488 1+8 089				

#### Common > ANOVA > Bayesian ANOVA

#### Model Comparison - meangen

Models	P(M)	P(M data)	BFM	8F10	error %
Null model	0.200	1.195e-9	4.780e-9	1.000	
frequency	0.200	3.823e-10	1.529e-9	0.320	1.317e -5
sampling	0.200	0.309	1.792	2.590e+8	2.988e-14
frequency + sampling	0.200	0.255	1.367	2.132e+8	0.849
frequency + sampling + frequency * sampling	0.200	0.436	3.089	3.647e+8	1.130

#### Analysis of Effects - meangen

Effects	P(incl)	P(incl data)	BFInclusion
frequency	0.600	0.691	1.488
sampling	0.600	1.000	4.227e+8
frequency * sampling	0.200	0.436	3.089

## 2.3 Bayesian t-test



## Planned analysis #1:

## Null effect under category sampling?

id	frequency	sampling	meangen	smallworld	largeworld	property	category
1	smallworld	property	4.67	property		smallworld	
2	largeworld	property	4.83		property	largeworld	
3	largeworld	category	4.33		category		largeworld
4	largeworld	property	4.33		property	largeworld	
5	smallworld	category	6.00	category			smallworld
6	largeworld	property	5.17		property	largeworld	
7	smallworld	category	5.17	category			smallworld
8	smallworld	property	6.83	property		smallworld	
9	smallworld	property	3.17	property		smallworld	
10	largeworld	property	4.00		property	largeworld	
11	smallworld	category	5.67	category			smallworld
12	smallworld	category	5.50	category			smallworld
13	smallworld	property	3.00	property		smallworld	

#### Common > T-Test > Bayesian Independent Samples T-Test



### Common > T-Test > Bayesian Independent Samples T-Test



Bayesian independent Samples 1–1es	les T-Test	ent Sampl	epend	an Ind	Bayesia
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	BF01	error %
meangen	5.305	9.941e-7

## Planned analysis #2:

### large < small under property sampling

id	frequency	sampling	meangen	smallworld	largeworld	property	category
1	smallworld	property	4.67	property		smallworld	
2	largeworld	property	4.83		property	largeworld	
3	largeworld	category	4.33		category		largeworld
4	largeworld	property	4.33		property	largeworld	
5	smallworld	category	6.00	category			smallworld
6	largeworld	property	5.17	2011-0021	property	largeworld	
7	smallworld	category	5.17	category			smallworld
8	smallworld	property	6.83	property		smallworld	
9	smallworld	property	3.17	property		smallworld	
10	largeworld	property	4.00		property	largeworld	
11	smallworld	category	5.67	category			smallworld
12	smallworld	category	5.50	category			smallworld
13	smallworld	property	3.00	property		smallworld	

#### **Common > T-Test > Bayesian Independent Samples T-Test**



#### **Common > T-Test > Bayesian Independent Samples T-Test**



Bayesian Indep	endent Sample	es T-Test 🔻
	BF <sub>-0</sub>	error %
meangen	22.49	~3.662e -6
Note, For all te	sts the altern	ative hypothesis

specifies that group largeworld is less than group smallworld.

# 2.4 Bayesian regression



id	age	small	property	female	meangen
1	21	1	1	1	4.67
2	19	0	1	1	4.83
3	20	0	0	1	4.33
4	19	0	1	0	4.33
5	21	1	0	1	6.00
6	31	0	1	1	5.17
7	21	1	0	1	5.17
8	24	1	1	1	6.83
9	19	1	1	1	3.17
10	20	0	1	0	4.00
11	21	1	0	1	5.67
12	20	1	0	1	5.50
13	20	1	1	0	3.00
14	19	0	1	1	4.83
16	21	1	0	0	3.83
18	24	1	0	1	4.50
19	20	1	0	0	3.50
20	20	1	1	0	5.50

tutedata5.csv

#### **Common > Regression > Bayesian Linear Regression**



#### **Common > Regression > Bayesian Linear Regression**

Models	P(M)	P(M data)	BFM	BF10	error %
property	0.063	0.326	7.243	1.000	
small + property	0.063	0.234	4.572	0.717	0.012
age + small + property	0.063	0.151	2.663	0.463	0.012
age + property	0.063	0.141	2.457	0.432	0.012
property + female	0.063	0.050	0.791	0.154	0.013
small + property + female	0.063	0.043	0.678	0.133	0.012
age + small + property + female	0.063	0.030	0.471	0.093	0.013
age + property + female	0.063	0.025	0.392	0.078	0.012
Null model	0.063	6.055e -10	9.083e-9	1.860e-9	0.012
age	0.063	3.438e -10	5.157e-9	1.056e-9	0.012
small	0.063	1.745e -10	2.617e-9	5.358e-10	0.012
age + small	0.063	1.697e -10	2.546e-9	5.211e-10	0.013
female	0.063	1.023e -10	1.535e-9	3.142e-10	0.012
age + female	0.063	7.768e-11	1.165e-9	2.385e-10	0.014
age + small + female	0.063	4.557e-11	6.835e-10	1.399e-10	0.012
small + female	0.063	4.301e-11	6.451e-10	1.321e-10	0.015

#### Model Comparison - meangen 🔻

#### **Common > Regression > Bayesian Linear Regression**

#### Analysis of Effects – meangen ▼

Effects	P(incl)	P(incl data)	BFInclusion
age	0.500	0.347	0.532
small	0.500	0.458	0.845
property	0.500	1.000	6.402e+8
female	0.500	0.149	0.175

## 2.5 Bayesian contingency tables



id	age	small	property	female	meangen
1	21	1	1	1	4.67
2	19	0	1	1	4.83
3	20	0	0	1	4.33
4	19	0	1	0	4.33
5	21	1	0	1	6.00
6	31	0	1	1	5.17
7	21	1	0	1	5.17
8	24	1	1	1	6.83
9	19	1	1	1	3.17
10	20	0	1	0	4.00
11	21	1	0	1	5.67
12	20	1	0	1	5.50
13	20	1	1	0	3.00
14	19	0	1	1	4.83
16	21	1	0	0	3.83
18	24	1	0	1	4.50
19	20	1	0	0	3.50
20	20	1	1	0	5.50

#### **Common > Frequencies > Bayesian Contingency Tables**



#### **Common > Frequencies > Bayesian Contingency Tables**

¥	Statistics
San	npling
0	Poisson
C	Joint multinomial
C	Indep. multinomial, rows fixed
C	Indep. multinomial, columns fixed
C	Hypergeometric (2x2 only)
Hyp	oothesis
C	Group one ≠ Group two
C	Group one > Group two
0	Group one < Group two
Bay	es Factor
0	BF <sub>10</sub>
C	BF <sub>01</sub>
C	Log(BF <sub>10</sub> )

#### **Common > Frequencies > Bayesian Contingency Tables**

Bayesian Contingency Tables 🔻

	prop			
small	0	1	Total	
0	73	71	144	
1	62	78	140	
Total	135	149	284	

bayesian contingency rables rests	yesian C	ontingency	Tables	Tests
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	Value
BF01 joint multinomial	2.540
N	284

# 2.6 Bayesian binomial test



spin	outcome
1	red
2	red
3	red
4	black
5	red
6	red
7	red
8	black
9	red
10	red

#### **Common > Frequencies > Bayesian Binomial Test**

🚴 spin	\delta outcome	ок
Test value: 0.5		
Hypothesis	Plots	
O ≠ Test value	Prior and posterior	
> Test value	Additional info	
🔘 < Test value	Sequential analysis	
Bayes Factor	Prior	
O BF10	Beta prior: parameter a	1
BF01	Beta prior: parameter b	1
Log(BF10)		

D	escriptives	Common	A Reg	vession Frequ	encies	Factor *				
	🌲 spin	eutcome		Results						
1	1	red		Results	5					
2	2	red	Bavesian Binomial Test							
3	3	red	buyesian binomiar rest							
4	4	black		Bayesian Bin	omial Test					
5	5	red			Level	Counts	Total	Proportion	BF10	
6	6	red		outcome	black	2	10	0.200	2.069	
7	7	red		Note. Propor	tions tested	o against value	E 0.5.	0.000	2.009	
8	8	black								
	9	red								
9	-									



### Wait... we got 1.87 for this Bayes factor and JASP says 2.07



### It's just an approximation error... if we use finer-grained approximation to "continuous numbers" we get 2.05

# 2.7 Beyond basics



... to be added at a later stage!

